Introduction

The second assignment for 600.112: Introduction to Programming for Scientists and Engineers is meant to really get you going with programming in Python. However, it does not yet cover any specific application area in science or engineering, it is only about programming. We will focus on drawing things on the screen using Python’s Turtle Graphics module to make this journey more entertaining.

There are three things to do: First you’ll write a program that draws a number of basic geometric shapes on the screen. Second you’ll write a program that will plot one period of the sine function from $-\pi$ to $\pi$. Third you’ll write a program that will plot a parametric curve where the $x$ and $y$ positions are derived from a single parameter in an interesting way.

There are detailed submission instructions on Piazza which you should follow to the letter! You can lose points if you create more work than necessary for the graders by not following the instructions.

1 Geometric Shapes (20%)

The first program you will write draws a number of simple geometric shapes: triangles, squares, and heptagons (the seven-sided regular polygon). Please call your program shapes.py and nothing else! Figure 1 shows what the output of your program will look like.

For this and the following programs, you will need to use functions and for loops as discussed in lecture. This is in addition to your basic understanding of expressions and variables. We will, however, lead you through the problems rather slowly and with a lot of advice on how to proceed, so you should be okay as long as you follow along diligently.
Before you can do anything else, you’ll need a very basic first version of your program that sets up the turtle module and properly waits for the user to close the window. Here is what that first version could look like:

```python
import turtle
def main():
    turtle.setup()
    turtle.done()
main()
```

This version will literally do nothing but open the turtle window, wait for the user to close the window, and exit. Once you have this, you can write your first function, probably the one to draw triangles. As you can see from Figure 1, you will have to draw triangles of different sizes, so your function should take a size parameter. After deciding these two things, you can add the function and some code to test it to your program:

```python
import turtle
def triangle(size):
    for _ in range(3):
        turtle.forward(size)
        turtle.left(120)
def main():
    turtle.setup()
    triangle(100)
turtle.done()
main()
```

When you run this version of the program, you should see a single triangle on the screen. While that’s a far cry from the final image you’re supposed to draw, it’s certainly progress!

The process we just illustrated, starting with a very simple program and testing it, adding a little bit of code and testing again, adding a little more code and testing again, etc. is very important. If you sit down and write code for two hours before ever testing your program, you will be overwhelmed by all the things that are going wrong. If instead you write code for only two minutes and test your program again, there is be much less code that can go wrong, and therefore it will be a lot easier for you to correct your mistakes. Always program in baby steps!

Now that you have a working triangle function, you can write and test the square function in exactly the same manner. We’ll leave the function empty in the following, the code is almost identical to triangle anyway.

This brings us to the third shape you need to draw, the heptagon. We could write a function that draws exactly a heptagon, but you’ve probably noticed by now that the main difference between all regular polygons is (a) how many lines to draw and (b) how much to turn left between each line. So instead of writing a heptagon function, let’s write a polygon function that can draw any regular polygon we desire. Obviously it’s not enough to tell the polygon function how big of a polygon to draw,

```python
import turtle
def triangle(size):
    for _ in range(3):
        turtle.forward(size)
        turtle.left(120)
def main():
    turtle.setup()
    triangle(100)
turtle.done()
main()
```

If you run this version, it will behave just like the first one; however, you can now be sure that you didn’t make a mistake in defining the function and calling it. The next step is to develop the body of the triangle function. Drawing a triangle requires that we move forward three times and turn left three times. We should move forward each time by size; we need to turn left each time by 120 degrees (why?). So one step in the process of drawing a triangle is to (a) move forward and (b) turn; we need to perform this step three times, so we put it inside a for loop:
we also have to tell it how many sides the polygon is supposed to have:

```python
import turtle
def triangle(size):
    for _ in range(3):
        turtle.forward(size)
        turtle.left(120)
...
def polygon(size, sides):
    pass
def main():
    turtle.setup()
    triangle(100)
    square(100)
    polygon(100, 7)
turtle.done()
main()
```

Looking back at the structure of our previous functions, it should be clear that the for loop in polygon has to run sides times: it ran three times for a triangle, four times for a square, so it has to run seven times for a heptagon, five times for a pentagon, and so on. The angle by which we turn after each line has to depend on the number of sides as well: If we add up the angles for the triangle and the square, we get 360 each time; so for an arbitrary polygon, the angle should be 360/n where n is the number of sides. So here we go:

```python
import turtle
def triangle(size):
    for _ in range(3):
        turtle.forward(size)
        turtle.left(120)
...
def polygon(size, sides):
    for _ in range(sides):
        turtle.forward(size)
        turtle.left(360.0/sides)

def main():
    turtle.setup()
    triangle(100)
    square(100)
    polygon(100, 7)
turtle.done()
main()
```

Note that it is very important that we write 360.0 and not just 360 (why?).

At this point we can draw all the required shapes, so what we have left to do is draw them multiple times, in different colors, and at different positions. Colors are the easiest thing to get a handle on, so let’s start there. The turtle module provides a color function that we can use: we simply say turtle.color("red") for example. Figure 1 indicates that we need red, green, and blue for triangles, squares, and heptagons respectively, so we change our code as follows:

```python
import turtle
...
def main():
    turtle.setup()
    turtle.color("red")
    triangle(100)
    turtle.color("green")
    square(100)
    turtle.color("blue")
    polygon(100, 7)
turtle.done()
main()
```

Drawing each shape repeatedly at different sizes is obviously something a for loop can do. Each shape already takes a size parameter, so all we need to know is what sizes we are supposed to draw them at. Let’s say we want to draw each shape at a size of 10, 30, 50, and 70, so each shape gets drawn four times. Creating a for loop that goes through these values requires that we use the three-argument form of the
range function: range(10, 71, 20) Remember that the lower bound is inclusive while the upper bound is exclusive; if we would use 70 instead of 71, the value 70 itself wouldn’t be included. Let’s write a separate function for drawing our set of four triangles as follows:

```python
import turtle

def triangles():
    for size in range(10, 71, 20):
        triangle(size)

def main():
    turtle.setup()
    turtle.color("red")
    triangles()
    turtle.color("green")
    square(100)
    turtle.color("blue")
    polygon(100, 7)
    turtle.done()

main()
```

Following this example, you can write the functions squares and heptagons to draw four of each of those shapes, so our main becomes this:

```python
import turtle

def main():
    turtle.setup()
    turtle.color("red")
    triangles()
    turtle.color("green")
    square(100)
    turtle.color("blue")
    polygon(100, 7)
    turtle.done()

main()
```

Before calling squares we move it to the right by the same amount, and then again to the right by the same amount before calling heptagons. Done!

2 Plotting Sines (40%)

The second program you will write plots one period of the sine function from $-\pi$ to $\pi$. Please call your program sine.py and nothing else! Figure 2 shows what the output of your program will look like.

You already know how to switch colors from the first program you wrote, so drawing the axes in blue and the sine curve in red should not be a problem. Here is some basic code to get you started (with the important functions missing of course):

```python
import turtle

def main():
    turtle.setup()
    turtle.color("red")
    triangles()
    turtle.color("green")
    squares()
    turtle.color("blue")
    heptagons()
    turtle.done()

main()
```

Now we’re pretty close to what the program is supposed to draw, all that’s left is to move the turtle with the pen up before we draw each set of shapes. The turtle starts at position (0, 0) after setup, so we start by moving it a decent amount to the left before we draw the triangles:
Instead of using `forward` and `left` to draw a certain line, it is more convenient for this program to be able to draw a line from the current position of the turtle to an arbitrary position on the screen. Luckily the `turtle` module has a `goto` function that does exactly that: If we are currently at position (-10, 10) and call `turtle.goto(100, 100)`, the turtle will draw a line (provided the pen is down!) from (-10, 10) to (100, 100)!

Let’s attack the axes we need to draw first. Both axes should go from -200 to 200 in the respective coordinate, and they should meet at (0, 0). Instead of writing the code for this twice, let’s assume that there is a function `line(x1, y1, x2, y2)` that gets the x and y coordinates of two points to draw a line between (from the first point to the second point). If we have such a function, we can write the `axes` function as follows:

```python
import turtle

def axes():
    line(-200, 0, 200, 0)
    line(0, -200, 0, 200)

def main():
    turtle.color("blue")
    axes()

    turtle.color("red")
    plot()
    turtle.done()

main()
```

You’ll have to write the `line` function in terms of the `up`, `down`, and `goto` functions of the `turtle` module to finish drawing the axes.

Drawing the sine wave itself requires a `for` loop to create the successive x values that we want to calculate the sine for. You can access the sine function using `turtle.sin(x)` where x is measured in radians, not degrees. If you look at Figure 2, we are obviously plotting the sine function between \(-\pi\) and \(\pi\); however, in terms of screen coordinates, \(-\pi\) is actually at -150 on our x-axis. Also, the result of `turtle.sin(x)` is always between -1 and 1 but the extrema in terms of screen coordinates are -100 and 100. So you will have to scale the arguments to the sine function (as well as the result you get back from it) suitably to make the plot have the right dimensions on the screen.

Set up the `for` loop inside your `plot` function using `range(-150, 151, 5)` as your range. So the first value will be -150, the next will be -145, the next will be -140, and so on, up to 150 at the other end. Inside the `for` loop, calculate the actual
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x value for the sine function from these values; you’ll have to divide and multiply with the correct factors to make this happen. When you get back the result, multiply it by the correct factor to make the extrema of the sine $-100$ and $100$. How do you achieve the plot itself? Just use the `goto` function in the right way!

3 Plotting Parametrics (40%)

The third program you will write plots a parametric curve, a notion we’ll explain briefly below. Please call your program `parametric.py` and nothing else! Figure 3 shows what the output of your program will look like.

When we talk about “plotting a function” we usually think of something like $y = \sin x$ where we determine the value for $y$ from $x$ and then plot the coordinates $(x, y)$. A parametric curve of the kind shown in Figure 3 works a little differently: there is some parameter $t$ and both the $x$ and the $y$ coordinate we plot depend on $t$. For example, here is the parametric curve you are supposed to plot:

\[
\begin{align*}
    x &= \sin 2t \\
    y &= \cos 5t
\end{align*}
\]

The values for $t$ range from $0$ to $2\pi$. Since we cannot write a `for` loop using floating point numbers, you’ll have to write the range using integers. Use this one: `range(0, int(2*turtle.pi*1000), 10)` So instead of going from $0$ to $2\pi$ we go from $0$ to $2000\pi$ in terms of integer values; to get the correct floating point value, you’ll have to divide $t$ by $1000$ before you use it in the parametric equations. Also, you need to scale the resulting $x$ and $y$ values by $150$ to get the plot from Figure 3. Enjoy!

Bonus Curve

Parametric curves are fun because they can produce lots of interesting shapes. The parametric curve from above could, for example, be used in a game to move alien space ships across the screen in an interesting pattern. Figure 4 shows a parametric curve that could be useful for a love letter instead. The equation for the “heart curve” is as follows:

\[
\begin{align*}
    x &= 16 \sin^3 t \\
    y &= 13 \cos t - 5 \cos 2t - 2 \cos 3t - \cos 4t
\end{align*}
\]

Once your program works for the basic parametric curve, it will be able to plot all of them. So feel free to play around and try to find a curve you like!